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Locating Leaks in Underground Water Pipes Using the Complex Cepstrum

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Abstract: It is possible to detect the presence of leaks in underground water pipes by measuring, at remote locations such as hydrants, the noise or vibration caused by the leak. The time delay of the leak noise reaching the different sensors can be computed using the cross correlation, and with knowledge of the wavespeed in the pipe, the location of the leak may be pinpointed. This paper presents a new technique for leak detection which employs the cepstrum, rather than the cross correlation, for estimation of the delay time. The delay time manifests as a series of peaks in the cepstrum, rather than a single peak in the correlation, allowing a more robust estimate. A number of cepstrum formulations are presented which are derived from correlation estimators, and it is found that the time delay information is actually contained in the phase component of the cross spectrum. Based on this, a phase cepstrum estimator is developed.

Keywords: leaks, underground water pipes, cepstrum, cross correlation.

1 Introduction

Recently there has been a renewed focus on minimising the amount of water lost to leaks in metropolitan water supplies. Typically there are two stages involved in finding these leaks; firstly to find the sectors of the pipe network that contain leaks and secondly to locate the leaks more precisely by conducting a detailed leak detection survey. This second stage is the focus of this investigation, and this is where vibro-acoustic techniques are often employed (see e.g. [1] for a thorough overview). Currently there are three methods used in the water industry that are aimed at finding the precise location of leaks; listening devices (listening sticks and ground microphones), noise loggers, and noise correlators.

Listening devices are the oldest form of technology and probably started early last century by augmenting the operators' listening ability using some simple mechanical means of amplifying the leak noise relative to other sounds. This principle remains the same in modern day instruments; however electronics and more sophisticated transducers have replaced previous methods. Noise loggers and noise correlators are more recent developments, having only existed in the last 25 years. Noise loggers can indicate sections of pipe that contain leaks based on either attended monitoring or unattended night-time monitoring using multiple transducers that can be scanned from a remote location such as from a vehicle. These devices search for and detect the constant noise that is characteristic of a water leak against a background of intermittent noises such as those generated by non-constant water usage and environmental influences.

Leak noise correlators work by detecting the signal from a leak using two transducers located on valves or hydrants either side of a suspected leak. The cross-correlation function is used together with known or assumed information about the speed at which the leak noise propagates to quantify the distance of the leak from the two transducers.

Listening devices are useful only at the final stage of leak location when deciding precisely where to excavate and are the most manually intensive means of leak detection. Noise loggers sound like an attractive form of technology as they permit simultaneous measurement on a larger scale than other methods, however, noise loggers have been found to be 3 times less efficient than acoustic surveys and failed to detect approximately 40% of the leaks found in detailed listening surveys [2]. The apparent efficiency gain from the ability to remotely survey multiple pipe sections simultaneously is eroded by the high failure rate associated with this method. Of the three acoustic leak detection methods, noise correlators have been found to be the most efficient and most accurate [3].

The principle behind correlation based leak detection is relatively simple [4, 5]. As fluid escapes from a pipe, it emits sound which travels down the pipe and can be used not only to detect the presence of a leak, but also its location. Sensors are located at access points to the pipe either side of the suspected leak location, as shown in Figure 1.

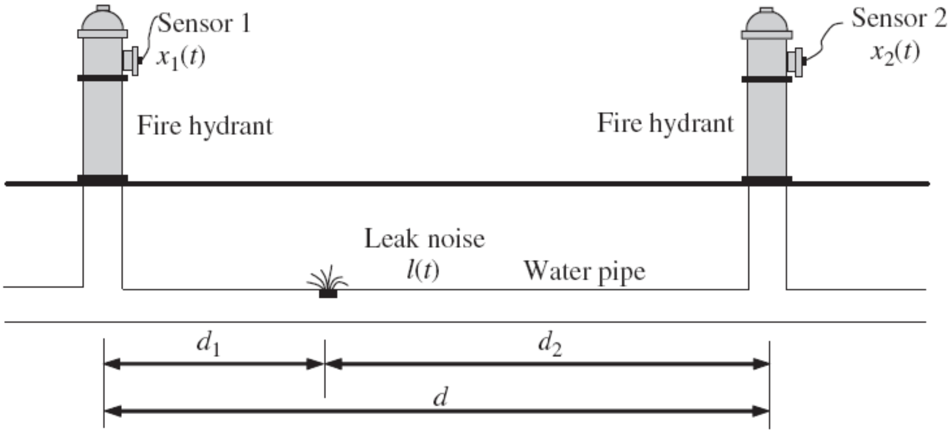

Figure 1 Leak detection using sensors on external fittings (from [5])

The cross correlation between the two measured signals, x_1 and x_2, is calculated, and exhibits a peak at a time lag corresponding to the difference in arrival time of the leak signal between the two sensors. The sound attenuates as it travels along the pipe; far more so in the increasingly common plastic pipes than in their metal forebears. For this reason, leak detection in plastic pipes corresponds to a worse case scenario, and will be considered in this investigation.

The strengths of the correlation technique lie in its effectiveness, simplicity, relative lack of expense compared with some other methods, and its portability, with the commercial products being easily man-portable. One of the main weaknesses of the technique is the requirement for *a priori* knowledge of the pipe system under test. The operator must know the general layout of the pipe network in order to accurately estimate the distance between the sensors, and also the pipe material which will affect the propagation speed. Such information is potentially unreliable, as network drawings and records of pipe repairs may not have been kept up to date. Other environmental factors can also reduce the accuracy of prediction, such as temperature [6] which effects the elastic properties of the pipe wall. This may have implications if the wavespeed is not measured, but rather sourced from a reference table.

Some of the more quantifiable weaknesses of the correlation technique relate to the ability of the operator (or software) to identify a clear peak in the cross correlation. This problem has many facets, including signal processing, which was addressed in this investigation by proposing that the cepstrum be employed, rather than the cross correlation, for the estimation of the delay time.

2 Theoretical Overview

The location of the leak in the pipe, relative to one of the sensors, can be estimated from:-

$$d_1 = \left(d - c\tau_{delay}\right)/2 \qquad (1)$$

where c is the wave speed in the pipe and τ_{delay} is the delay time, traditionally estimated using the cross correlation, but which can also be found from the cepstrum.

The complex cepstrum [7] is obtained from:-

$$c(\tau) = \Im^{-1}\left[\log\left(S_{xy}(\omega)\right)\right] \qquad (2)$$

where τ is quefrency*, $\Im^{-1}$ denotes the inverse Fourier transform, and $S_{xy}(\omega)$ is the cross spectrum between signals *x* and *y*. The complex cepstrum is a real function but is so-named because it derives from a complex spectrum. Similarly, the phase cepstrum is obtained from:-

$$c_p(\tau) = \Im^{-1}\left[\log\left(\phi_{xy}(\omega)\right)\right] \qquad (3)$$

where now only the phase of the cross spectrum ϕ_{xy} is used. In the normal definition, the phase is unwrapped to a continuous function of frequency.

2.1 Numerical Simulation

This investigation made use of a numerical simulation where the leak was assumed to be white noise. The sensor signals x_1 and x_2 were calculated using the frequency response function of the pipe which, at a distance *x* along from the leak (and assuming the use of accelerometers), is given by [4, 5]:-

$$H(\omega, x) = (i\omega)^2 A e^{-i\omega x/c} e^{-\omega\beta x} \qquad (4)$$

where β is a measure of the loss in the pipe wall, $A = a^2/(Eh)$ where *a* is the mean pipe radius, *h* is the pipe wall thickness and *E* is the Young's modulus of the pipe wall.

2.2 Cepstrum Time Delay Estimators

In [5], Gao *et al* apply a number of different weightings which filter the cross spectrum from which the cross correlation is estimated. By taking the log of these filtered cross spectra before the inverse Fourier transformation, an equivalent cepstrum estimator can be derived. These estimators are presented here, where the name of each estimator is taken from its cross correlation equivalent, but with a prefix 'c' to denote the cepstrum domain.

The simplest estimator is the cBCC (cepstrum Basic Cross Correlation) which is obtained from:-

$$cBCC(\omega) = \Im^{-1}\left\{\log\left(S_{x_1x_2}(\omega)\right)\right\} \qquad (5)$$

It does not include any pre-filtering or pre-whitening and so offers a benchmark against which the other estimators can be assessed.

In contrast, the cSCOT (cepstrum Smoothed Coherence Transform) estimator employs the square root of the coherence to attenuate frequency regions where noise is present and pre-whitening by dividing by the cross spectrum magnitude:-

$$cSCOT(\omega) = \Im^{-1}\left\{\log\left(\left(\gamma_{x_1x_2}(\omega) S_{x_1x_2}(\omega)\right) / \left|S_{x_1x_2}(\omega)\right|\right)\right\} \qquad (6)$$

The cPHAT (cepstrum PHAse Transform) estimator is similar to the cSCOT estimator but without the frequency weighting offered by the coherence term in (6). Indeed, and as explained by Gao, the cPHAT estimator reduces to the cSCOT estimator in the noise free case, i.e. when $\gamma = 1$.

$$cPHAT(\omega) = \Im^{-1}\left\{\log\left(S_{x_1x_2}(\omega) / \left|S_{x_1x_2}(\omega)\right|\right)\right\} \qquad (7)$$

The cWIENER estimator instead applies the frequency weighting but avoids the pre-whitening. In the noise free case, it reduces to the cBCC estimator.

* Quantities in the cepstrum domain are named for their frequency domain analogue, but with the first syllable reversed. Hence *ceps*trum derives from spectrum, *quef*rency from frequency and *rah*monic from harmonic.

$$cWIENER\left(\omega\right)=\Im^{-1}\left\{\log\left(\gamma_{x_1x_2}^{2}\left(\omega\right)S_{x_1x_2}\left(\omega\right)\right)\right\} \tag{8}$$

The cML (cepstrum Maximum Likelihood) estimator is derived to minimise the variance if the signals are random Gaussian, and is given by:-

$$cML\left(\omega\right)=\Im^{-1}\left\{\log\left(\left(\gamma_{x_1x_2}^{2}\left(\omega\right)S_{x_1x_2}\left(\omega\right)\right)/\left(\left[1-\gamma_{x_1x_2}^{2}\left(\omega\right)\right]\left|S_{x_1x_2}\left(\omega\right)\right|\right)\right)\right\} \tag{9}$$

The ROTH estimator attempts to estimate the impulse response of the system by removing the effects of the input through normalising by one of the response autospectra. As the input cannot be measured however, an impulse response estimate cannot be obtained. The cROTH estimator is obtained from:-

$$cROTH\left(\omega\right)=\Im^{-1}\left\{\log\left(S_{x_1x_2}\left(\omega\right)/S_{x_1x_1}\left(\omega\right)\right)\right\} \tag{10}$$

These estimators were used to calculate the time delay using the numerical model of the pipe given in (4), where the sensors were placed 25m and 75m from the “leak”. The results of this simulation are presented below.

3 Results

While the difference between the cross correlation and cepstrum estimators is simply one of taking the log of the spectrum, this has a profound effect on their utility for leak detection. While the time delay manifests as a peak in the cross correlation displaced from the origin, it appears as a train of rahmonics in the cepstrum, each located at an exact integer multiple of the delay time, thereby providing not one but several estimates.

The cross correlation and cepstrum estimators calculated for the simulated system are shown in Figure 2. The time delay of approximately 104ms was correctly estimated in all cases due to the relatively idealised assumption of uncorrelated noise. It can be envisaged however, that even if the first rahmonic in the cepstrum estimators was polluted by noise, a higher rahmonic could be employed to provide a more robust estimate. If the cross correlation was heavily polluted by noise however, no such option would be available.

It is interesting to note that the cPHAT estimator provides the clearest estimate of the delay time in Figure 2, and this is investigated further in the next section.

4 Phase Cepstrum Time Delay Estimator

The purpose of these pre-whitening/weighting operations applied by Gao was to reduce the effects of noise and/or to sharpen the peak associated with the delay time. The PHAT estimator was obtained by weighting the cross spectrum by its magnitude, as seen in (7). This function reduces to a complex exponential term containing the phase of the cross spectrum and the PHAT correlation estimator is obtained by the inverse Fourier transformation of this complex exponential:

$$PHAT\left(\tau\right)=\Im^{-1}\left\{S_{x_1x_2}\left(\omega\right)/\left|S_{x_1x_2}\left(\omega\right)\right|\right\}=\Im^{-1}\left\{\left(\left|S_{x_1x_2}\left(\omega\right)\right|e^{-j\omega\tau_{delay}}\right)/\left|S_{x_1x_2}\left(\omega\right)\right|\right\}=\Im^{-1}\left\{e^{-j\omega\tau_{delay}}\right\} \tag{11}$$

The PHAT correlation estimator, in the noise free case, is therefore a delta function at the delay time τ_{delay}, which is also noted by Gao. The cPHAT estimator however is obtained by taking the log of the exponential function in (11), from which the phase is obtained:

$$\log\left(e^{-j\omega\tau_{delay}}\right)=-j\omega\tau_{delay} \tag{12}$$

If the phase is unwrapped to give the normal phase cepstrum it is a ramp function without periodicity, but if is not unwrapped, then it jumps 2π every $1/\tau_{delay}$ Hz, as shown in Figure 3. All the cepstrum estimators have thus been obtained by inverse Fourier transformation of the corresponding function with wrapped phase. The cPHAT estimator from the simulation above, presented again in Figure 3, shows fifteen clear rahmonics (abbreviated in the figure to nine for clarity). Even though noise would affect the exact locations of the $2p$ jumps, the mean spacing (and thus quefrency) would be correct.

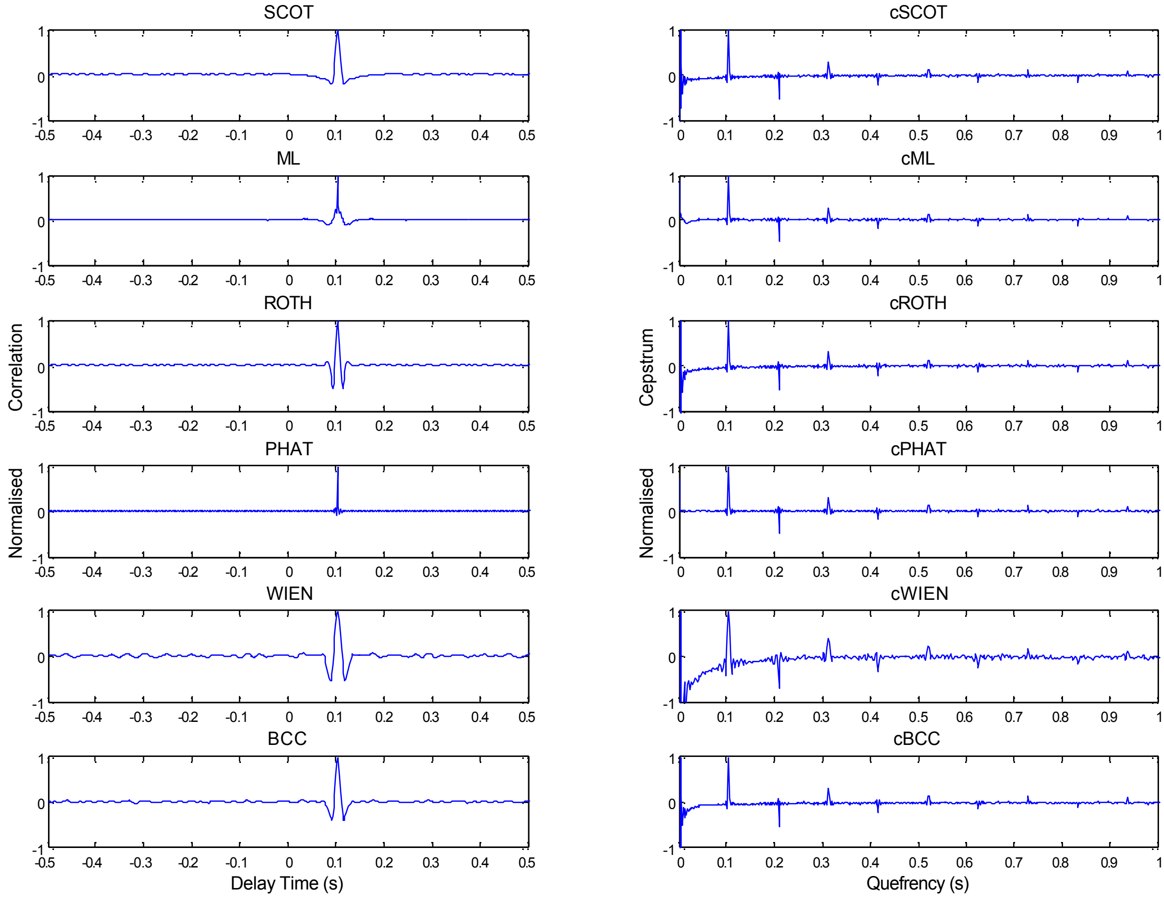

Figure 2 Comparison of correlation based estimators (left) and cepstrum based estimators (right)

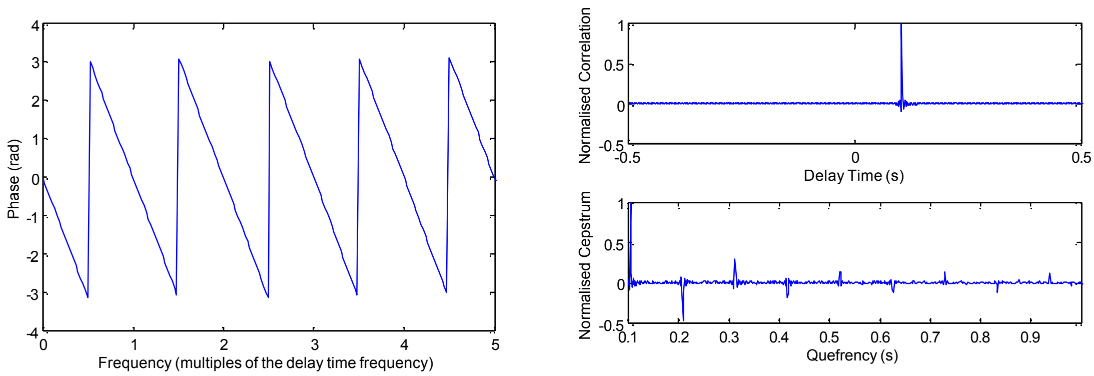

Figure 3 Wrapped phase of the cross spectrum (left) and the PHAT (top right) and cPHAT (bottom right) estimators

As the PHAT and cPHAT have revealed, the time delay information is contained solely in the phase of the cross spectrum, which is unaffected by the amplitude modifications. This casts doubt on the effectiveness of several of the estimators presented in [5] which address only the magnitude of the cross spectrum. An advantage may still be obtained however by the band pass filtering effect of weighting functions based on the coherence, when used in combination with the phase cepstrum, such as the ML estimator.

4 Conclusions and Further Work

This paper has discussed the use of the cepstrum as an alternative to the traditional cross correlation for locating leaks in underground water pipes. The cepstrum has the advantage that the delay in arrival time of the leak noise signal at two sensors manifests as a train of rahmonics, any or all of which can be employed in the calculation of the leak location. In contrast, the cross correlation offers a single peak displaced from the origin.

In an idealised simulation with uncorrelated noise, both the correlation and cepstrum estimators correctly indicated the delay time. It was discovered however, that the delay time information is contained in the phase of the cross spectrum prompting the development of a phase cepstrum estimator. In addition to the results presented here, a number of further developments are planned, including those outlined below.

4.1 Measured Leak Noise Signals

Further to the successful simulation of the phase cepstrum estimator is the application to measured leak noise signals from a real water pipe. We are currently investigating the possibility of acquiring such signals from an industry partner and hope to present these results in the near future.

4.2 Automatic Leak Detection

The nature of the rahmonics opens the possibility of incorporating a robust automatic leak detection algorithm. Such an algorithm might proceed by firstly identifying the *r*th rahmonic as a local peak where $t > n$. In this case, 'n' is an arbitrary quefrency above which the rahmonics are clearly identifiable from any noise. The (*r+1*)th rahmonic could be identified in a similar fashion, where 'n' is now the quefrency of the *r*th rahmonic. This would proceed until a clear rahmonic could no longer be identified. The time delay could then be estimated from $t_{delay} = t_m / m$ where *m* is the rahmonic number. It is intended to apply such an estimator to the measured leak signals and further refinements would be made on the basis of its performance.

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